CSC 258 midterm

27 February 2002, 11:00

Name (underline surname):

Student number:

Tutorial section:

No aids permitted, but there is a list of algebraic identities attached.
Total: 40 marks.
Time allotted: 45 minutes.

Since time is short, be careful not to get stuck on one question to the exclusion of others. Not everyone will necessarily be able to finish this test within the 45 minutes. The amount of marks or answer-space allotted does not indicate how long it will take you to complete the question, nor does the size of the answer-space indicate the size of the correct answer.

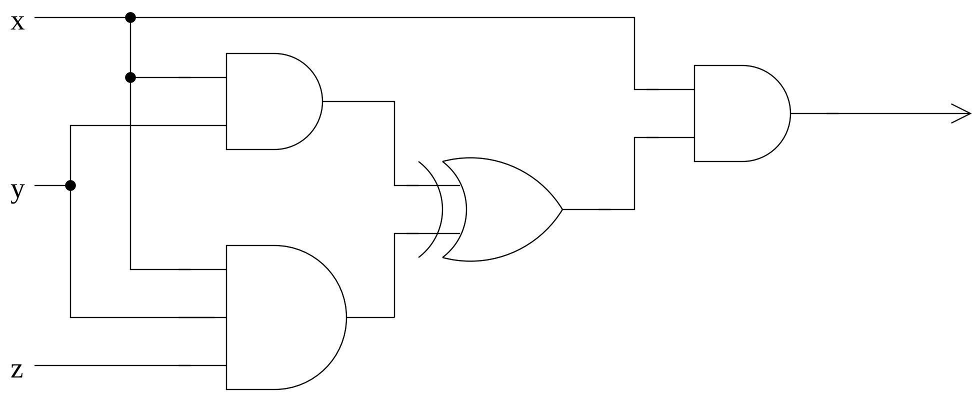
Answer *all* questions. Answer questions in the space provided.

Do not open this booklet until you are instructed to.

Do not write anything in the following table:

	value	mark
1	10	
2	10	
3	10	
4	10	
total	40	

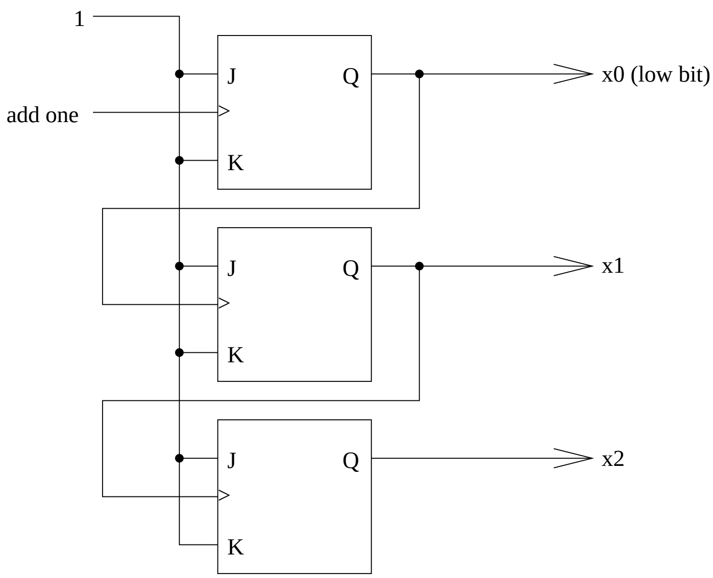
1. [10 marks]
a) What function does the following logic gate diagram compute?



b) Simplify this formula (using any appropriate technique).

c) Draw a logic gate diagram for your simplified formula.

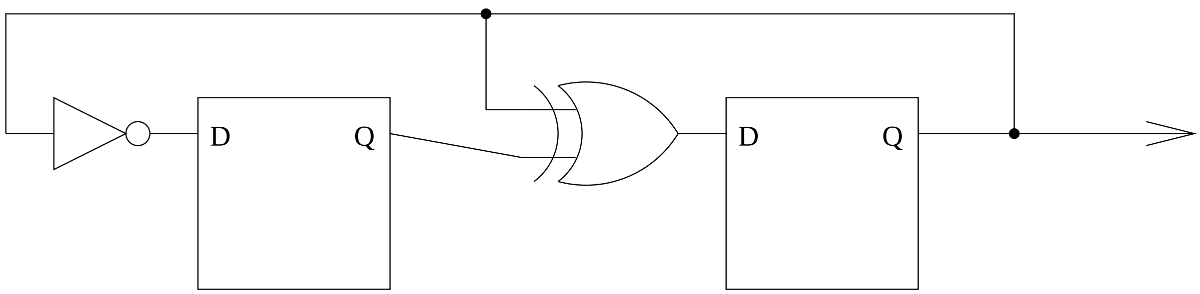
2. [10 marks]
Here is a three-bit ripple counter.



Draw a three-bit counter with *two* inputs: an ‘‘add one’’ input such as above, and a ‘‘complement’’ input which toggles all three bits (e.g. 010 will become 101).

You can assume that these two inputs will never be 1 at the same time.

3. [10 marks]
What is the output sequence of the following ‘‘counter’’, after it gets established in its cycle?



4. [10 marks]
a) Here are some four-bit addition problems, some of which will overflow. Complete the addition operation and state the base-ten value of the operands and the result. There are two examples.

0001	0111	0100	0101	1000	0001
+0010	+0010	+0011	+0011	+1000	+1011
-----	-----	-----	-----	-----	-----
0011	1001				
1+2 = 3	7+2 = −7 (overflow)				

b) Show that overflow is *not* the same as carry out (from the entire addition, i.e. c_n) by using one of the examples above. You needn’t show your reasoning; just identify the example, state whether or not overflow occurs in that addition, and state whether or not carry-out occurs.
(The standard overflow formula is $c_n \oplus c_{n-1}$, although you probably don’t need to know this to do this question.)

Extra space if needed
(you must write ‘‘see page 5’’ in the usual answer space for the given question)

Some Boolean algebra identities

identity laws:

$$a \cdot 1 = a$$
$$a + 0 = a$$

idempotence:

$$aa = a$$
$$a + a = a$$

base laws:

$$a \cdot 0 = 0$$
$$a + 1 = 1$$

excluded middle:

$$a + \bar{a} = 1$$

non-contradiction:

$$a \cdot \bar{a} = 0$$

double-negation:

$$\bar{\bar{b}} = b$$

exclusive-or definition:

$$a \oplus b = a\bar{b} + \bar{a}b$$

commutative:

$$ab = ba$$
$$a + b = b + a$$
$$a \oplus b = b \oplus a$$

associative:

$$(ab)c = a(bc)$$
$$(a + b) + c = a + (b + c)$$
$$(a \oplus b) \oplus c = a \oplus (b \oplus c)$$

distributive:

$$a(b + c) = ab + ac$$
$$a + bc = (a + b)(a + c)$$

de Morgan's laws:

$$\overline{a + b} = \bar{a}\bar{b}$$
$$\overline{(ab)} = \bar{a} + \bar{b}$$

etc

absorption:

$$a(a + b) = a$$
$$a + ab = a$$
$$a + \bar{a}b = a + b$$

no name:

$$ab + a\bar{b} = a$$